Review Exercises

In Exercises 1–4, write the first five terms of the sequence. (Assume that n begins with 1.)

1. $a_n = 2 + \dfrac{6}{n}$

2. $a_n = \dfrac{5n}{2n - 1}$

3. $a_n = \dfrac{72}{n!}$

4. $a_n = n(n - 1)$

In Exercises 5–8, use a graphing utility to graph the first 10 terms of the sequence.

5. $a_n = \frac{3}{2}n$

6. $a_n = 4(0.4)^{n-1}$

7. $a_n = \dfrac{3n}{n + 2}$

8. $a_n = 5 - \dfrac{3}{n}$

In Exercises 9–12, use sigma notation to write the sum.

9. $\dfrac{1}{2(1)} + \dfrac{1}{2(2)} + \dfrac{1}{2(3)} + \cdots + \dfrac{1}{2(20)}$

10. $2(1^2) + 2(2^2) + 2(3^2) + \cdots + 2(9^2)$

11. $\dfrac{1}{2} + \dfrac{2}{3} + \dfrac{3}{4} + \cdots + \dfrac{9}{10}$

12. $1 - \dfrac{1}{3} + \dfrac{1}{9} - \dfrac{1}{27} + \cdots$

In Exercises 13–20, find the sum.

13. $\displaystyle\sum_{i=1}^{6} 5$

14. $\displaystyle\sum_{k=2}^{5} 4k$

15. $\displaystyle\sum_{j=1}^{4} \dfrac{6}{j^2}$

16. $\displaystyle\sum_{i=1}^{8} \dfrac{i}{i + 1}$

17. $\displaystyle\sum_{k=1}^{10} 2k^3$

18. $\displaystyle\sum_{j=0}^{4} (j^2 + 1)$

19. $\displaystyle\sum_{n=0}^{10} (n^2 + 3)$

20. $\displaystyle\sum_{n=1}^{100} \left(\dfrac{1}{n} - \dfrac{1}{n + 1} \right)$

In Exercises 21–24, write the first five terms of the arithmetic sequence.

21. $a_1 = 3, \ d = 4$

22. $a_1 = 8, \ d = -2$

23. $a_4 = 10, \ a_{10} = 28$

24. $a_2 = 14, \ a_6 = 22$

In Exercises 25–28, write the first five terms of the arithmetic sequence defined recursively. Determine the common difference and write the nth term of the sequence as a function of n.

25. $a_1 = 35 \qquad a_{k+1} = a_k - 3$

26. $a_1 = 15 \qquad a_{k+1} = a_k + \frac{5}{2}$

27. $a_1 = 9 \qquad a_{k+1} = a_k + 7$

28. $a_1 = 100 \qquad a_{k+1} = a_k - 5$

In Exercises 29 and 30, write an expression for the nth term of the arithmetic sequence and find the sum of the first 20 terms of the sequence.

29. $a_1 = 100, \ d = -3$

30. $a_1 = 10, \ a_3 = 28$

In Exercises 31–34, find the sum.

31. $\displaystyle\sum_{j=1}^{10} (2j - 3)$

32. $\displaystyle\sum_{j=1}^{8} (20 - 3j)$

33. $\displaystyle\sum_{k=1}^{11} (\tfrac{2}{3}k + 4)$

34. $\displaystyle\sum_{k=1}^{25} \left(\dfrac{3k + 1}{4} \right)$

35. Find the sum of the first 100 positive multiples of 5.

36. Find the sum of the integers from 20 to 80 (inclusive).

37. *Job Offer* The starting salary for a job is $34,000 with a guaranteed salary increase of $2250 per year. Determine (a) the salary during the fifth year and (b) the total compensation through five full years of employment.

38. *Baling Hay*   In the first two trips baling hay around a large field, a farmer obtains 123 bales and 112 bales, respectively. Because each round gets shorter, the farmer estimates that the same pattern will continue. Estimate the total number of bales made if there are another six trips around the field.

In Exercises 39–42, write the first five terms of the geometric sequence.

39. $a_1 = 4$, $r = -\frac{1}{4}$

40. $a_1 = 2$, $r = 2$

41. $a_1 = 9$, $a_3 = 4$

42. $a_1 = 2$, $a_3 = 12$

In Exercises 43–46, write the first five terms of the geometric sequence defined recursively. Determine the common ratio and write the nth term of the sequence as a function of n.

43. $a_1 = 120$ $a_{k+1} = \frac{1}{3}a_k$

44. $a_1 = 200$ $a_{k+1} = 0.1a_k$

45. $a_1 = 25$ $a_{k+1} = -\frac{3}{5}a_k$

46. $a_1 = 18$ $a_{k+1} = \frac{5}{3}a_k$

In Exercises 47 and 48, write an expression for the nth term of the geometric sequence and find the sum of the first 20 terms of the sequence.

47. $a_1 = 16$, $a_2 = -8$

48. $a_1 = 100$, $r = 1.05$

In Exercises 49–54, find the sum.

49. $\displaystyle\sum_{i=1}^{7} 2^{i-1}$

50. $\displaystyle\sum_{i=1}^{5} 3^{i-1}$

51. $\displaystyle\sum_{i=1}^{\infty} \left(\frac{7}{8}\right)^{i-1}$

52. $\displaystyle\sum_{i=1}^{\infty} \left(\frac{1}{3}\right)^{i-1}$

53. $\displaystyle\sum_{k=1}^{\infty} 4\left(\frac{2}{3}\right)^{k-1}$

54. $\displaystyle\sum_{k=1}^{\infty} 1.3\left(\frac{1}{10}\right)^{k-1}$

In Exercises 55 and 56, use a graphing utility to find the sum.

55. $\displaystyle\sum_{i=1}^{10} 10\left(\frac{3}{5}\right)^{i-1}$

56. $\displaystyle\sum_{i=1}^{25} 100(1.06)^{i-1}$

57. *Depreciation*   A company buys a machine for $120,000. During the next 5 years it will depreciate at a rate of 30% per year. (That is, at the end of each year the depreciated value will be 70% of what it was at the beginning of the year.)

   (a) Find the formula for the nth term of a geometric sequence that gives the value of the machine t full years after it was purchased.

   (b) Find the depreciated value of the machine at the end of five full years.

58. *Total Compensation*   A job pays a salary of $32,000 the first year. During the next 39 years, suppose there is a 5.5% raise each year. What would the total salary be over the 40-year period?

59. *Compound Interest*   A deposit of $200 is made at the beginning of each month for 2 years in an account that pays 6%, compounded monthly. What is the balance in the account at the end of 2 years?

60. *Compound Interest*   A deposit of $100 is made the beginning of each month for 10 years in account that pays 6.5%, compounded monthly. What is the balance in the account at the end of 10 years?

In Exercises 61–64, use mathematical induction to prove the formula for every positive integer n.

61. $1 + 4 + \cdots + (3n - 2) = \dfrac{n}{2}(3n - 1)$

62. $1 + \dfrac{3}{2} + 2 + \dfrac{5}{2} + \cdots + \dfrac{1}{2}(n + 1) = \dfrac{n}{4}(n + 3)$

63. $\displaystyle\sum_{i=0}^{n-1} ar^i = \dfrac{a(1 - r^n)}{1 - r}$

64. $\displaystyle\sum_{k=0}^{n-1} (a + kd) = \dfrac{n}{2}[2a + (n - 1)d]$

In Exercises 65–68, evaluate $_nC_r$ or $_nP_r$. Use the $_nC_r$ or the $_nP_r$ feature of a calculator to verify your answer.

65. $_6C_4$ **66.** $_{10}C_7$

67. $_8P_5$ **68.** $_{12}P_3$

In Exercises 69–74, use the Binomial Theorem to expand the binomial. Simplify your answer. (Remember that $i = \sqrt{-1}$.)

69. $\left(\dfrac{x}{2} + y\right)^4$ **70.** $(a - 3b)^5$

71. $\left(\dfrac{2}{x} - 3x\right)^6$ **72.** $(3x + y^2)^7$

73. $(5 + 2i)^4$ **74.** $(4 - 5i)^3$

75. *Amateur Radio*   A Novice Amateur Radio license consists of two letters, one digit, and then three more letters. How many different licenses can be issued if no restrictions are placed on the letters or digits?

76. *Morse Code*   In Morse code, all characters are transmitted using a sequence of dits and dahs. How many different characters can be formed by a sequence of three dits and dahs? (Repetition is allowed. For example, dit-dit-dit represents the letter "s.")

77. *Matching Socks*   A man has five pairs of socks of which no two pairs are the same color. If he randomly selects two socks from a drawer, what is the probability that he gets a matched pair?

78. *Bookshelf Order*   A child returns a five-volume set of books to a bookshelf. The child is not able to read, and hence cannot distinguish one volume from another. What is the probability that the books are shelved in the correct order?

79. *Roll of the Dice*   Are the chances of rolling a 3 with one die the same as the chances of rolling a total of 6 with two dice? If not, which has the higher probability?

80. *Roll of the Dice*   A six-sided die is rolled six times. What is the probability that each side appears exactly once?

81. *Tossing a Coin*   Find the probability of obtaining at least one tail when a coin is tossed five times.

82. *Parental Independence*   In a survey, senior citizens were asked if they would live with their children when they reached the point of not being able to live alone. The results are shown in the figure. Suppose three senior citizens who cannot live alone are randomly selected. What is the probability that all three are not living with their children?

Would live with children **17%**

Don't know **3%**

Would not live with children **80%**
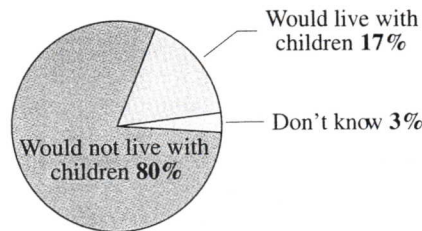
83. *Card Game*   Five cards are drawn from an ordinary deck of 52 playing cards. Find the probability of getting exactly two pairs. (For example, the hand could be A-A-5-5-Q or 4-4-7-7-K.)

84. *Data Analysis*   A sample of college students, faculty, and administration were asked whether they favored a proposed increase in the annual activity fee to enhance student life on campus. The results of the study are given in the table.

	Students	Faculty	Admin.	Total
Favor	237	37	18	292
Oppose	163	38	7	208
Total	400	75	25	500

A person is selected at random from the sample. Find the specified probability.

(a) The person is not in favor of the proposal.

(b) The person is a student.

(c) The person is a faculty member and is in favor of the proposal.